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A Computer Model of the House-Environment System

by

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INTRODUCTION

Historically, there has been a strong relation between the environment of a region and the type of dwelling man has constructed in that region (Aronin, 1953; Olgyay, 1963; Rapoport, 1969). Dwellings are meant in part to be comfortable places to live, and man like most other life, is comfortable only within certain ranges of temperature, humidity, wind speed, and exposure (Givoni, 1969; Landsberg, 1954; Terjung, 1968). The use of clothing enables man considerably more flexibility than most other life forms, yet in his dwellings man likes to be comfortable without the necessity of excessive amounts of clothing.

For reasons of economy and comfort, man throughout history has often optimized useful aspects of and minimized detrimental aspects of the external environment in the design of his houses. For example, a thick heavy roof and walls were incorporated into a design to retain the evening cold during the hot day, and insulation of walls and roof was used for economy in artificial heating. Climatic considerations are by no means the only determining factors of house form, other important ones being the available materials and cultural traditions (Rapoport, 1969).

We have presently reached a stage in architecture where considerations of the interaction between the environment and the house are for all practical purposes unnecessary. Every modern house has its electrical cord which supplies almost unlimited lighting, heating, and cooling. Sewer, water, and gas pipes supply or take away the other necessities. In transporting a house design from Los Angeles, California, to Anchorage, Alaska, the only major change may be the reversing of the relative strengths of the heater and air-conditioner.

This ignoring of the external environment seems dangerous. Recent results from theory in the atmospheric and earth sciences (Budyko, 1969, 1970, 1972; Faegre, 1972; SCEP, 1970; Sellers, 1969) indicate that the earth's global and regional climates are not as stable as might be hoped. The indications are that it is well within man's power to unexpectedly and irreversibly change climate in directions that would at least threaten our continued existence. Extrapolating to the year 2000, the heat that man produces in housing, industry, and transportation is not expected to be responsible (by direct heating) for substantial climatic change. However, the by-products of energy consumption, especially particulate, chemical, and water vapor pollutions, are of major concern. On a global scale these strongly influence the amount of shortwave radiation that is absorbed and longwave radiation that is emitted from the earth-system. On the regional scale these pollutants directly threaten life.

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There seem to be at least three clear reasons to work toward housing which interacts responsibly with the environment. First, given that housing in the US consumes 21% of the nation's energy (Kupperman, 1972) it is clear that the environmental problems are at least partly related to housing. Until we clean up our energy sources and reduce their pollutants, it is obviously in our interest to reduce the energy requirements of houses. Second, with the growing concern about limited natural resources there is likely to be governmental action encouraging and requiring energy conservation in housing (Kupperman, 1972). The proposed encouragement would initially be limited to what is termed "leak plugging", i.e. increased insulation, but the possibility of drawing on the external environment is not ruled out. Lastly, we often make the aesthetic judgement that we wish our house to bear a climatic relation to the area within which it exists. Furthermore, it may be that a house which makes use of energy "leaks" to and from its environment might prove educationally valuable in keeping its occupants conscious of the larger environmental problems.

While attempting to explore solutions to the house-environment climate system, the author developed a simple model with which one might become familiar with the flows of energy between a house and its environment. The flows of energy and the sources and sinks of energy of the house system are identified and put into a mathematical form. Although a more realistic model would include at least indoor humidity and wind velocity, in this model the primary measure of indoor climate is temperature. The degree of complexity these other variables would add to the theory makes it impossible, for purely practical reasons, to presently include them.

With the model one can experiment with changes in house color, orientation and size of windows, and building materials. Given the simplicity of the model it is not expected to give completely realistic solutions, yet we expect it to indicate in a general sense the outcome or effect of change in house design.

In its present form, the model is probably most useful as an educational tool for the architecture student. It is the author's intention to refine the model to a point where it may be useful to the practicing architect.

THE MODEL

THE STEADY STATE APPROXIMATION OF A BOX WITH WINDOWS

The basic equation of the model is the equation of conservation of energy. In this case it is simply the statement that the amount of energy, O, flowing from the house must equal the amount of energy, I, flowing into the house plus the amount of energy, H, generated within the house, i.e.

$$O = I + H \tag{1}$$

The terms O and I represent energy which flows between the external and the internal environment. The term H represents what are called sources and sinks, within the house. They add or take away heat without any intermediate transfer process.

In this model all energy leaving the house, O, will be by heating of the outside air. All energy flowing into the house, I, is from either the sun or in cases of variable outside air temperature heat may be convected to the house, in which case it will be a negative component in the O term.

Consider the i^{th} wall or ceiling (the floor will be considered as perfectly insulated). We require that the energy C_i convected from the outside house walls to the outside air must equal the energy S_i radiated to the wall from the sun, plus a term F_i which represents heat flowing out through the wall and has contributions from a heater, an air-conditioner, and heat that has flowed in another wall to now flow out this one, i.e.

$$C_i = S_i + F_i \qquad (i = 1, 2, \ldots, n) \tag{2}$$

There will be n of these equations, one for each wall or ceiling. For the simple box (no heat flow through the floor) $n = 5$.

There is a condition on the F_i. The sum of them over i must equal H of eqn (1). That is, each F_i is in part energy that has flowed in one wall (e.g. the wall facing the sun) and now flows out another wall (e.g. a wall not facing the sun) i.e.

$$F_1 + F_2 + \ldots + F_n = H \tag{3}$$

It should be clear that $H > 0$ for heating, $H < 0$ for cooling (air-conditioning), and $H = 0$ for the absense of both. We now determine formulas for C_i, S_i and F_i.

In general we consider heat as flowing proportionally to the temperature gradient. So heat F_i flowing out through the wall is written:

$$F_i = K_f\,(T_{Ii} - T_{Oi})\,R_i \tag{4}$$

where T_{Oi} is the outside temperature of the i^{th} wall. T_{Ii} is the inside temperature of the i^{th} wall, K_f is the thermal conductance of the wall, and R_i is the area of the wall.

Heat C_i flowing to the outside air will be proportional to the temperature difference between the outside of the wall and the outside air, i.e.

$$C_i = K_c\,(T_{Oi} - T_A)\,R_i \tag{5}$$

where T_A is the outside air temperature, and K_c is a surface coefficient governing the heat flow. In practice, K_c is dependent on wind velocity and surface roughness.

Heating of the i^{th} wall from the sun is expressed:

$$S_i = (1 - W_i)\,Q\,(1 - A_i)\,\cos(z_i)\,\cos(h_i)\,R_i \tag{6}$$

where Q is the incident radiation, A_i is the albedo, z_i and h_i are the sun's angles with respect to the wall, W_i is the fraction of the wall that is window. The windows are considered as suitably insulated so as not to alter the terms C_i and F_i.

Substituting eqn (4), (5) and (6) into eqn (2) yields:

$$\begin{aligned} K_c(T_{Oi} - T_A)\,R_i &= (1 - W_i)Q(1-A_i)\cos(z_i)\cos(h_i)R_i \\ &\quad + K_f(T_{Ii} - T_{Oi})R_i \qquad (i = 1, 2, \ldots, n) \end{aligned} \tag{7}$$

with the condition (eqn (3)) that

$$\sum_{i=1}^{n} K_f\,(T_{Ii} - T_{Oi})\,R_i = H \tag{8}$$

Further assuming that the inside walls of the house are at equal temperature, i.e.

$$T_{I1} = T_{I2} = \dots = T_{In} = T_I \tag{9}$$

this becomes:

$$\begin{aligned} K_c(T_{Oi} - T_A)R_i &= (1 - W_i)Q(1 - A_i)\cos(z_i)\cos(h_i)R_i \\ &\quad + K_f(T_I - T_{Oi})R_i \end{aligned} \tag{10}$$

with the constraint that:

$$K_f R_I \sum_{i=1}^{n} R_i = K_f \sum_{i=1}^{n} T_{Oi}R_i + H \tag{11}$$

or where:

$$\sum_{i=1}^{n} R_i = R_T \tag{12}$$

$$T_I = \frac{\sum_{i=1}^{n} T_{Oi}R_i}{R_T} + \frac{H}{K_f R_T} \tag{13}$$

Eqn (13) can now be put into eqn (11) to yield:

$$\begin{aligned} K_c(T_{Oi} - T_A)R_i &= (1 - W_i)Q(1 - A_i)\cos(z_i)\cos(h_i)R_i \\ &\quad - K_f T_{Oi}R_i + K_f R_i \left(\frac{\sum_{i=1}^{n} T_{Oi}R_i}{R_T} + \frac{H}{K_f R_T}\right) \end{aligned} \tag{14}$$

Expanding the equation for i=1 and n=5, and collecting terms of T_{Oi} yields:

$$\begin{aligned} K_c T_A R_1 &+ (1 - W_1)Q(1 - A_1)\cos(z_1)\cos(h_1)R_1 + \frac{HR_1}{R_T} \\ &= (K_c R_1 + K_f R_1 - \frac{K_f(R_1)^2}{R_T})\, T_{O1} - (\frac{K_f R_1 R_2}{R_T})\, T_{O2} \\ &\quad - \dots - (\frac{K_f R_1 R_5}{R_T})\, T_{O5} \end{aligned} \tag{15}$$

The direct internal heating, H, comes from several sources, the principle two being sun coming through the windows, H_{sun}, and heat from electric or other powered devices, H_{man}, i.e.

$$H = H_{sun} + H_{man} \tag{16}$$

The contribution of the sun to direct internal heating is the sum of the energy through all the windows, i.e.

$$H_{sun} = \sum_{i=1}^{n} W_i R_i Q \cos(z_i)\cos(h_i) \tag{17}$$

For n=5 we see there are five linear equations similar to eqn (15). The problem then is to simultaneously solve the five linear equations. This is not difficult with the use of a computer. Before doing so, we will add a heat storage term to the model to make it time dependent.

THE TIME DEPENDENT APPROXIMATION OF A BOX WITH WINDOWS

In allowing the model to be time dependent, we recognize that the climate of the house is affected by more than the instantaneous parameters. The indoor temperature does not suddenly drop when the sun goes behind a cloud. The major term involved in shifting to a time dependent theory is the stored heat, L, of the house. We will write it in the simple form, for the i^{th} wall:

$$L_i = M_i C_L (\frac{T_{Oi} + T_I}{2}) \qquad (18)$$

where M_i is the mass, C_L the heat capacity of the material, and $(\frac{T_{Oi} + T_I}{2})$ represents the average temperature of the wall.

During a time increment Δt, T_{Oi} and T_I will change and this will cause the absorption or release of ΔL_i to the house, i.e.

$$\Delta L_i = M_i C_L (\frac{T_{Oi}^* - T_{Oi} + T_I^* - T_I}{2}) \qquad (19)$$

where T^* is the temperature a Δt previous.
Letting $S_i' = S_i + \Delta L_i$ and replacing S_i by S_i' in eqn (6) we get the resultant equation analogous to eqn (15):

$$K_c T_A R_1 + (1 - W_1) Q (1 - A_1) \cos(z_1) \cos(h_1) R_1 + \frac{H R_1}{R_T}$$
$$+ M_1 C_L (\frac{T_{O1}^* - T_{O1} + T_I^* - T_I}{2}) = (K_c R_1 + K_f R_1$$
$$- \frac{K_f (R_1)^2}{R_T}) T_{O1} - (\frac{K_f R_1 R_2}{R_T}) T_{O2}$$
$$- \dots - (\frac{K_f R_1 R_5}{R_T}) T_{O5} \qquad (20)$$

Expanding T_I and collecting terms we get:

$$K_c T_A + (1 - W_1) Q (1 - A_1) \cos(z_1) \cos(h_1) R_1 + \frac{H R_1}{R_T}$$
$$+ M_1 C_L \Big((\frac{T_{O1}^* + T_I^*}{2}) - \frac{H}{2 K_f R_T}) = (K_c R_1 + K_f R_1 - \frac{K_f (R_1)^2}{R_T}$$
$$+ \frac{M_1 C_L}{2} + \frac{M_1 C_L R_1}{2 R_T}) T_{O1} + (- \frac{K_f R_1 R_2}{R_T} + \frac{M_1 C_L R_2}{2 R_T}) T_{O2}$$
$$+ \dots + (- \frac{K_f R_1 R_5}{R_T} + \frac{M_1 C_L R_5}{2 R_T}) T_{O5} \qquad (21)$$

Eqn (21) is in the final form we want. Given T_{Oi}^* and T_I^*, the temperatures of the house a Δt previous, and allowing any change in the parameters over time (allowing the sun's angles, wind speed, etc. to change), we can determine the new temperatures by solving the set of equations.

SOLVING BY DIGITAL COMPUTER

The five linear equations can be written in the form:

$$\sum_{j=1}^{5} X_{ij} T_{Oj} = Y_i \qquad (i=1, 2, \ldots, 5) \qquad (22)$$

By expansion of eqn (14), for i=2, i=3, i=4, and i=5, into equations similar to that of eqn (22), it is easily found that in general:

$$X_{ij} = -\frac{K_f R_i R_j}{R_T} + \frac{M_i C_L R_j}{2 R_T} \qquad \text{if } i \neq j$$

$$X_{ij} = R_i \left(K_c + K_f\left(1 - \frac{R_i}{R_T}\right) + \frac{M_i C_L}{2 R_T}\right) + \frac{M_i C_L}{2} \qquad \text{if } i = j$$

$$Y_i = R_i\left((1 - W_i)\cos(z_i)\cos(h_i)Q(1 - A_i) + K_c T_A + \frac{H}{R_T}\right)$$

$$+ M_i C_L \left(\left(\frac{T_{Oi}^* + T_I^*}{2}\right) - \frac{H}{2 K_f R_T}\right) \qquad (23)$$

The following is a summary of the computer program that solved the set of equations. The values of the constants, the parameters, and initial conditions T_{Oi}^* and T_I^* are read into the computer. The five equations are determined in the form of eqn (22). They are solved by the use of the subroutine SIMQ of the Scientific Subroutine Package available from the IBM Corporation. This determines a new set of temperatures T_{Oi} which are printed out and from which T_I is determined (eqn (13)). We skip on to the next time increment by replacing the old T_{Oi}^* and T_I^* with the new values, changing any of the parameters that are time dependent, and repeating the solution process. This cycle is repeated a predetermined number of times, at which time the program terminates.

COMPUTER SIMULATION OF A PHYSICAL MODEL

To see if the model's predictions are at least in some cases within reasonable approximation of the physical counterpart, the computer model was set to simulate a physical model prepared by Givoni and Hoffman (Givoni, 1969; Givoni and Hoffman, 1968). The model of Givoni and Hoffman had external dimensions 115 cm x 153 cm and internal height 157 cm, walls of 15 cm thickness lightweight concrete whitewashed on the outside, and a window of glass 66 cm x 101 cm in the south facing wall. They then experimented with four different internal walls and compared results. For our simulation we used their data for an internal wall of 7 cm thickness lightweight concrete separated from the external wall by a 1 cm air space.

Using formulas and specific values from other parts of Givoni's (1969) book, the values of conductivity (K_f), specific heat (C_L), surface coefficient (K_c) (assumed average of 200 cm/sec wind speed), surface albedo (A_i), and mass (M_i) were determined. An average value of Q was assumed. The values of these parameters are given in Table 1. Data on the sun's angles corresponding to the period of the test was determined from solar charts in Givoni (1969).

TABLE 1.

K_f	thermal conductance of wall, 2.77×10^{-5} cal/(cm^2. sec . ^oC)
K_c	external surface coefficient, 4.78×10^{-2} cal/(cm^2. sec . ^oC)
Q	incident radiation, 0.011 cal/(cm^2 . sec)
A	albedo, 0.88
M	mass of walls determined from specific weight of lightweight concrete (specific wt = 0.600 g/cm^3)
C_L	specific heat, 0.25 cal/(g . ^oC)

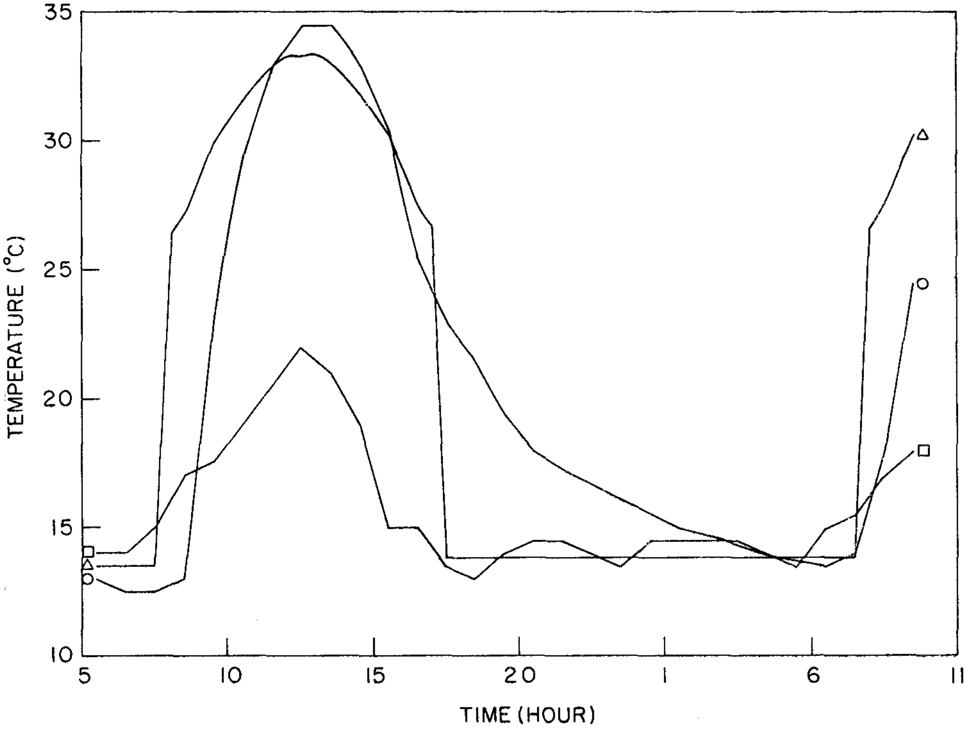

Fig. 1. Graphs of temperature versus time: □ the outside air temperature, O the inside air temperature of the physical model by Givoni and Hoffman (Givoni, 1969), and △ the predicted inside wall temperature using the computer model with the parameters given in Table 1.

The predicted inside temperatures are shown in Fig. 1, labeled △. The inside temperatures of the physical model are labeled O. The outside air temperatures are labeled □ . It is seen that the computer model roughly similates the physical model. It appears, however, that the computer model is too sensitive to internal heating by the sun and not sensitive enough to the external air temperature.

Altering the surface coefficient (K_c) and the thermal conductance of the wall (K_f), remedies this problem. Increasing K_c to $K_c = 9.56 \times 10^{-2}$ and $K_f = 6.00 \times 10^{-5}$ yields the inside temperatures labeled ▽ in Fig. 2. We find this to be in fact a quite good simulation of the physical model. Given the incompleteness of the necessary data from the physical model (cloudiness and windspeed were approximated), we do not know if the crudeness of the simulation shown in △ is due to theoretical problems or due to improper values of the parameters.

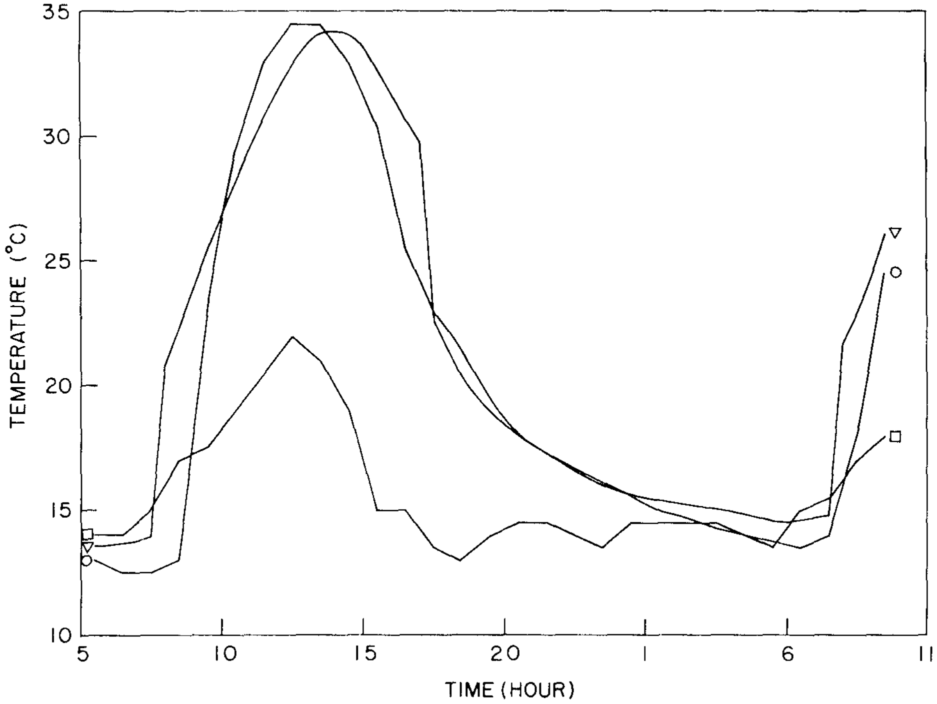

Fig. 2. Graphs of temperature versus time: □ the outside air temperature, O the inside air temperature of the physical model by Givoni and Hoffman (Givoni, 1969), and ▽ the predicted inside wall temperature using the computer model with altered values of $K_c = 9.56 \times 10^{-2}$ and $K_f = 6.00 \times 10^{-5}$.

A more complete set of experimental data is needed to determine which of these is actually the case. Given that a somewhat minor altering of the parameters K_c and K_f does yield a quite good simulation, ▽, it is believed that the computer model is basically sound.

CONCLUSION

The computer model is easily "played" with. Values of the wall albedoes are easily changed and their effect on indoor temperature quickly seen. A house design is easily transported from Los Angeles to Anchorage by a change in certain of the parameters. The design itself is easily altered by a change in other parameters. In using the computer model, one has the instant capability to experiment in the environmental design of housing. It is hoped that use of thise type of model might lead to better housing for modern man.

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ABSTRACT.- An energy-flow computer model of a house is developed which can be made to simulate the changes in temperature of a simple physical model during the 24-hr day. By adjusting parameters (albedo, sun's angles, conduction coefficients, wall dimensions, window dimensions, house orientation, internal heating or air-conditioning, insolation) the computer model is easily altered. It is presumably an easier model to experiment with than physical models. Experimentation with the computer model by architects should help them to maximize useful and minimize detrimental aspects of the external climate in the design of houses.

ZUSAMMENFASSUNG.- Ein Energiefluss Computer-Modell eines Hauses wird beschrieben, das zur Simulierung der Veränderungen in der Temperatur eines einfachen physikalischen Models während eines 24-Stunden Tages benutzt werden kann. Das Computer-Modell lässt sich leicht durch Anpassung der Parameter (Albedo, Sonnenstand, Leitungskoeffizient, Dimensionen der Wände und Fenster, Stellung des Hauses, Heizung oder Luftkonditionierung) verändern. Man kann damit vermutlich leichter experimentieren als mit physikalischen Modellen. Es sollte Architekten beim Entwurf von Häusern helfen, um wertvolle Aspekte des äusseren Klimas maximal auszunutzen und nachteilige auszuschalten.

RESUME.- On décrit ici un modèle, utilisable sur ordinateur, du flux d'énergie à l'intérieur d'une maison. Ce modèle permet de simuler les modifications de température d'un immeuble aux conditions physiques simples pendant les 24 heures de la journée. Ce modèle peut être modifié facilement en adaptant les paramètres (albédo, hauteur du soleil, coefficient de conductibilité, dimensions des parois et des fenêtres, orientation de l'immeuble, chauffage ou conditionnement de l'air). Il est plus facile d'expérimenter par cette méthode qu'en utilisant des modèles uniquement physiques. Cette nouvelle méthode devrait aider les architectes dans l'établissement de plans de maisons, en leur apportant de précieuses valeurs provenant du climat extérieur et en leur évitant par là même bien des déboires.